NOTE DUE DATES!

Closing **MONDAY**: HW_3C

Closing **FRIDAY**: HW_4A,4B,4C

Entry Task:

Let R be the region bounded by

$$y = \frac{1}{x^2} + \frac{1}{x}, y = 0, x = 1, x = 2.$$

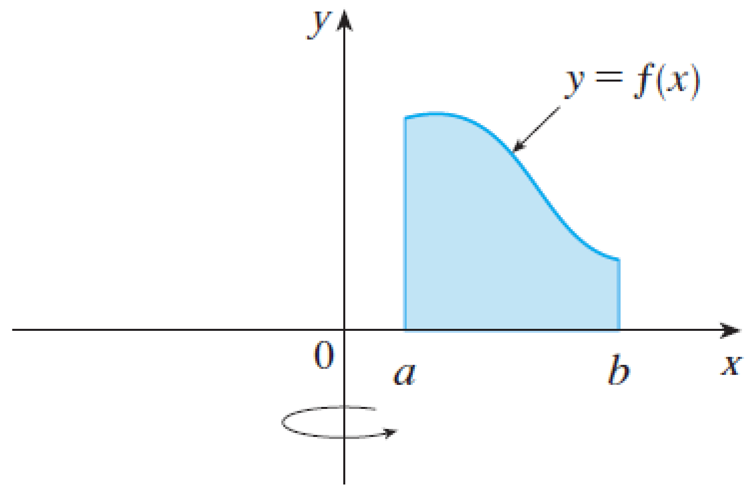
(a) Set up an integral for the volume of the solid obtained by rotating R about the x-axis.

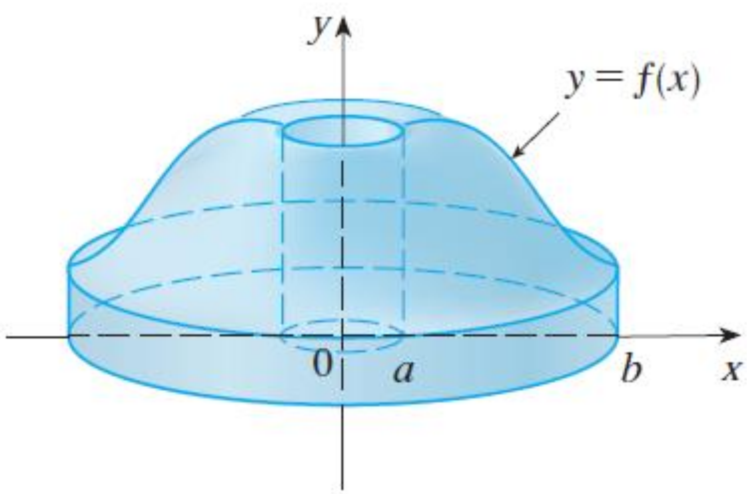
(b) Try to use cross-sectional slicing to set up an integral for the volume obtained by rotating R about the *y*-axis. Why is this difficult/messy?

6.3 Volumes Using Cylindrical Shells

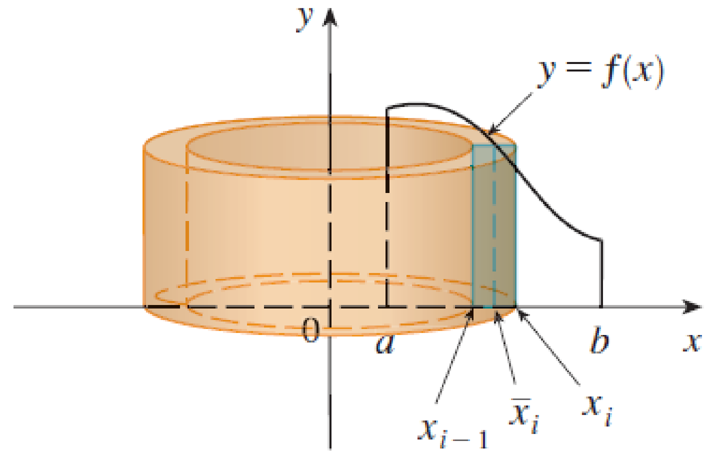
Visual Motivation:

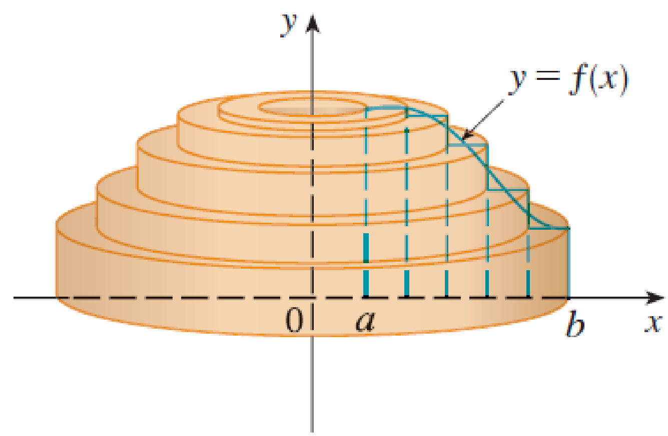
Consider the solid





We want to us “dx”, but that breaking the region into thin vertical subdivisions and rotating those gives a new shape, “cylindrical shells”





Derivation:

The pattern for the volume of one thin cylindrical shell is

VOLUME = 2π(radius)(height)(thickness)

= (surface area)(thickness)

Thus, if we can find a formula, $SA(x_i)$, for the surface area of a typical cylindrical shell, then

Thin Shell Volume ≈ $SA(x_i)\,\Delta x$,

$$\text{Total Volume} \approx \sum_{i=1}^{n} \text{SA}(\text{x}_i)\,\Delta\text{x}$$

$$\text{Exact Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} \text{SA}(\text{x}_i)\Delta\text{x}$$

$$\text{Volume} = \int_a^b SA(x)dx$$

$$= \int_a^b 2\pi(radius)(height)dx$$

Example:

Let R be the region bounded by

$$y = x^3, y = 4x,$$

$$\text{between x} = 1 \text{ and } x = 2.$$

1. Set up the integrals for the volume of the solid obtained by rotating R **about the y-axis**.
   (a) Using dy.
   (b) Using dx.

2. What changes if we rotate about the vertical line x = -2?

3. What changes if we rotate about the vertical line x = 3?

Volume using cylindrical shells

1. Draw a typical rectangle **parallel** to the axis of rotation.
   Label location (x or y) and the thickness (dx or dy) of a typical rectangle.

2. Draw a typical cylindrical shell
   Label everything in terms of the labeled variable.

3. Find the formula for the surface area of a typical shell:
   radius = ? (looks like *x*, *x-a* or *a-x*)
   height = ? (involves the functions)

4. Integrate!

$$\int_a^b 2\pi(\text{radius})(\text{height})(\text{dx or dy})$$

Volume using cross-sectional slicing

1. Draw a typical rectangle **perpendicular** to the axis of rotation. Label location (x or y) and the thickness (dx or dy) of a typical rectangle.

2. Draw a typical cross-section area.
   Label everything in terms of the labeled variable.

3. Find the formula for the cross-sectional area:

Disc: Area = $\pi(\text{radius})^2$

Washer: Area = $\pi(\text{outer})^2 - \pi(\text{inner})^2$

4. Integrate!

$$\int_a^b (\pi(\text{outer})^2 - \pi(\text{inner})^2)(\text{dx or dy})$$

Flow chart of all Volume Methods

Step 1: Choose the variable you want to use
(based on the region and the given equations)

Step 2: Draw typical rectangle based on the variable you chose which will either be perpendicular (disc/washer) or parallel (shells) to the axis of rotation.
Label location of rectangle and thickness.

Step 3: Perpendicular → *Cross-sections*: Find pattern for radius of disc/washers.
Parallel →*Shells*: Find pattern for radius and height of shells.

Step 4: Integrate the appropriate pattern as we have discussed.

If you still are having trouble seeing which variable goes with which method here:

Axis of rotation	**Disc/Washer**	**Shells**
x-axis (or any horizontal axis)	dx	dy
y-axis (or any vertical axis)	dy	dx